VIT BHOPAL

Term End Examinations – Interim Semester 2025-26 - December 2025	
Course code : **MAT3002**	Slot : **A21+ A22+ A23+ B14+ D21**
Course Title : **Applied Linear Algebra**	Time: **180 Minutes**
Date/Session : **26th Dec, 2025 / Session-I**	Max. Marks: **100**

KEEPING MOBILE PHONE/ANY ELECTRONIC GADGETS, EVEN IN 'OFF' POSITION, IS TREATED AS EXAM MALPRACTICE

General Instructions, if any:

1. **Non-programmable calculator is permitted: YES**
2. **Reference tables are permitted: NO**

PART – A

Answer ANY FIVE Questions (5 X 12 = 60 Marks)

S. No.		Questions	Marks
1	(a)	An instructor wants to make a question paper of 25 marks with 8 questions where each question is worth 2, 3, or 5 marks based on difficulty. He wants the number of 2-mark questions to be 1 more than the number of 5-mark questions. How many 2, 3, and 5 mark questions could be there? Solve by using Gauss-Elimination Method.	12
		(OR)	
	(b)	Solve the following system of linear equations by the Gauss-Jordan method: $x - 2y = -4$ $-5y + z = -9$ $4x - 3z = -10$	12
2	(a)	Let W be a subspace of $\mathbb{R}^3$ spanned by the following vectors $v_1 = (1,0,1), v_2 = (0,1,1), v_3 = (1,1,2), v_4 = (1,2,1), v_5 = (-1,1,-2)$ find the basis and dimension of W.	12
		(OR)	
	(b)	Find a basis for the row space, column space, and null space of $A = \begin{bmatrix} -1 & 0 & -1 & 2 \\ 2 & 0 & 2 & 0 \\ 1 & 0 & 1 & -1 \end{bmatrix}$.	12
3	(a)	Let $T: \mathbb{R}^3 \to \mathbb{R}^2$ be a linear transformation defined by $T(x, y, z) = (x - 2y, 2x + y)$ Find Range space, Null Space, Rank and Nullity.	12
		(OR)	
	(b)	Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ is a linear transformation defined by $T(x, y, z) = (3x, x - y, 2x + y + z)$. Is it invertible. If so find the rule of T^{-1} like T.	12
4	(a)	Use the Gram–Schmidt process to transform the basis vectors $u_1 = (1,0,1), u_2 = (1,0,-1), u_3 = (0,3,4)$ into an orthonormal basis under the standard Euclidean dot product.	12
		(OR)	
	(b)	For the weighted inner product $< u, v > = 4u_1v_1 + 5u_2v_2$; $u = (-4,6), v = (3,-5)$ and $w = (2,3)$, find the followings: **(i)** $< u, v >$ **(ii)** $< v, w >$ **(iii)** $\| v \|$ **(iv)** $d(u, w)$ **(v)** Verify the triangular inequality using the vectors u, v. **(vi)** Verify the Cauchy-Schwartz inequality using the vectors u, v.	12

5	(a)	Find QR decomposition of the matrix of $A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 2 \\ 1 & 1 & 3 \end{bmatrix}$	12
		(OR)	
	(b)	Find the SVD of $\begin{bmatrix} -2 & 2 \\ 1 & -1 \end{bmatrix}$	12
6	(a)	Find the Eigen values and Eigen vectors of $A = \begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$	12
		(OR)	
	(b)	Let $T: R^3 \to R^3$ be the linear transformation defined by $T(x, y, z) = (2x - 3y + 4z, 5x - y + 2z, 4x + 7y)$ Find the matrix of transformation T w.r.t. the basis $B = \{(1,1,0), (1,0,1), (0,1,1)\}$ and $B' = \{(1,1,1), (1,1,0), (1,0,0)\}$ where B is the basis of domain and B' is the basis of codomain.	12

PART – B

Answer ANY FIVE Questions (40 Marks)

7	Find the inverse of the following matrix by using elementary row operations $A = \begin{bmatrix} 1 & \text{[illegible]} & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$	8
8	Is $W = \{(x, y, z) \in \mathbb{R}^3 : x \geq 0, y, z \in \mathbb{R}\}$ a subspace of $\mathbb{R}^3$? Justify your answer. NO	8
9	Show that the mapping $T: \mathbb{R}^3 \to \mathbb{R}^3$ defined by $T(x, y, z) = (x - y + z, x + y - z, -x + y + z)$ is a linear transformation.	8
10	Check whether $< x, y > = y_1(x_1 + 2x_2) + y_2(2x_1 + x_2)$ is an Inner product or not for $x = (x_1, x_2)$ and $y = (y_1, y_2)$ for $\mathbb{R}^2$.	8
11	Find a least square solution of the system $Ax = b$ and also find the orthogonal projection of b onto the column space of A, where $A = \begin{bmatrix} 3 & -6 \\ 4 & -8 \\ 0 & 1 \end{bmatrix}, b = \begin{bmatrix} -1 \\ 7 \\ 2 \end{bmatrix}$	8
12	In the vector space $\mathbb{R}^3$ express the vector $(6,8,1)$ as a linear combination of vectors $(4,0,-1), (1,2,3), (-2,6,5)$. Justify your answer.	8